K_{sp}

What is the solubility of $FeCO_3$?

Solubility = MAXIMUM amount of a compound that can dissolve in water.

This is actually an equilibrium.

Equilibrium problems involve 3 parts:

1. Balanced equation
2. "K-equation"
3. ICE chart

What is the balanced equation for dissolving something?

$FeCO_{3\,(s)} \leftrightarrow Fe^{2+}_{(aq)} + CO_3^{2-}{}_{(aq)}$

What is the "K-equation"?

$K = [Fe^{2+}][CO_3^{2-}]$

The "K" is the PRODUCT of the SOLUBLE ions. Hence, this reaction is called a "solubility product".

$K_{sp} = [Fe^{2+}][CO_3^{2-}]$

$K_{sp}(FeCO_3) = 3.07 \times 10^{-11}$

What is the solubility of $FeCO_3$?

$K_{sp}(FeCO_3) = 3.07x10^{-11}$

$FeCO_{3\,(s)} \leftrightarrow Fe^{2+}{}_{(aq)} + CO_3{}^{2-}{}_{(aq)}$

I	S	0	0
C	-x	+x	+x
E	0.000000000001	x	x

$K_{sp} = 3.07x10^{-11} = [Fe^{2+}][CO_3{}^{2-}] = x*x$

$x = SQRT(3.07x10^{-11}) = 5.54x10^{-6}$ M

Clicker question

What is the solubility of $Ba_3(PO_4)_2$ at 298K?

$K_{sp}(Ba_3(PO_4)_2) = 6x10^{-39}$

A. $8x10^{-20}$ M
B. $2x10^{-8}$ M
C. $3x10^{-9}$ M
D. $9x10^{-9}$ M
E. $3x10^{-20}$ M

$Ba_3(PO_4)_2\ (s) \leftrightarrow 3\ Ba^{2+}\ (aq) + 2\ PO_4^{3-}(aq)$

I	S	0	0
C	-x	+3x	+2x
E	-	3x	2x

$K_{sp} = 6x10^{-39} = (3x)^3(2x)^2 = 27x^3*4x^2$

$5.56x10^{-41} = x^5$

$x = 8.89x10^{-9}\ M = 9x10^{-9}\ M$

More common units for solubility…

…are g/L.

If you wanted g/L

$$9\times10^{-9} M = \frac{9\times10^{-9} mol Ba_3(PO_4)_2}{L}\frac{602g}{1mol} = 5\times10^{-6} g/L$$

Precipitation Reaction

The reverse reaction:

Solubility Product:

$Ba_3(PO_4)_2$ (s) $\leftrightarrow$ 3 Ba^{2+} (aq) + 2 PO_4^{3-}(aq)

Precipitation:

3 Ba^{2+} (aq) + 2 PO_4^{3-}(aq) $\leftrightarrow$ $Ba_3(PO_4)_2$ (s)

It's just K "upside down"

How do you know if something precipitates?

$Ba_3(PO_4)_2\ (s) \leftrightarrow 3\ Ba^{2+}\ (aq) + 2\ PO_4^{3-}(aq)$

$K_{sp} = 6x10^{-39}$

What is the K_{sp}?

It's the limit on the amount of ions in solution.

$K_{sp} = [Ba^{2+}]^3[PO_4^{3-}]^2$

Remember our old friend "Q"?

What's Q?

Q is just the concentrations of products and reactants when you are NOT at equilibrium.

$K_{sp} = [Ba^{2+}]^3[PO_4^{3-}]^2 = 6x10^{-39}$

$Q = [Ba^{2+}]^3[PO_4^{3-}]^2 =$ any other number

Q is less than K means…

1. You are NOT at equilibrium.
2. You could dissolve more solid: the products (dissolved ions) are too small.

Q is more than K means…

1. You are NOT at equilibrium.
2. You have TOO MANY products (dissolved ions). They can't stay dissolved, they need to precipitate out!

A little precipitation question:

500 mL of 0.100 M $Fe(NO_3)_3$ is mixed with 250 mL of 0.100 M KOH. What, if anything, precipitates from the solution? What mass of precipitate is formed?

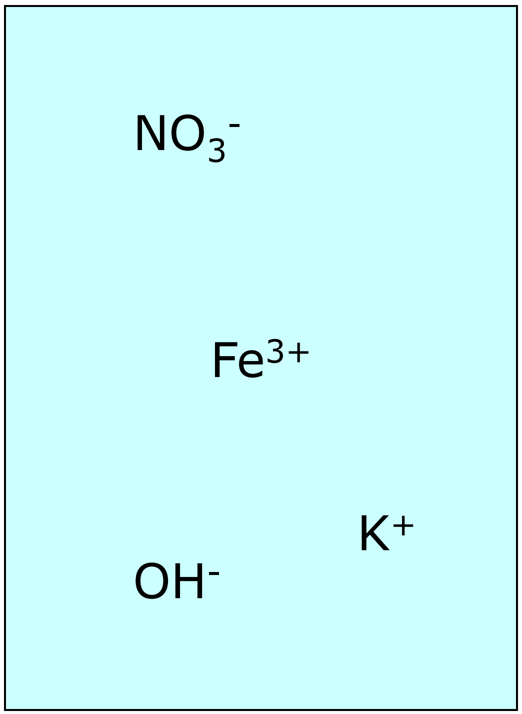
What COULD form…?

$KOH(s) \rightarrow K^+ (aq) + OH^-(aq)$

$Fe(NO_3)_3 \rightarrow Fe^{3+} (aq) + 3\ NO_3^- (aq)$

A beaker of KOH and $Fe(NO_3)_3$ has neither KOH nor $Fe(NO_3)_3$, it's all ions!

The 1st Rule of Chemistry…



Opposites attract!

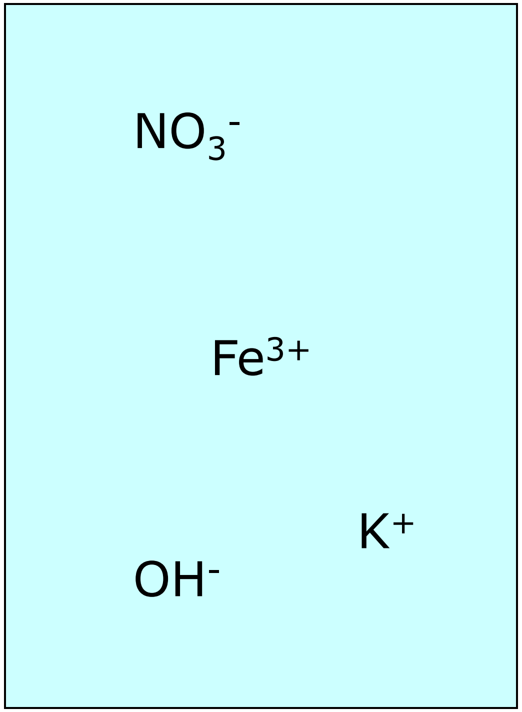
Positive ions like negative ions.

Negative ions like positive ions.

Postive ions hate positive ions.

Negative ions hate negative ions.

Only possible products are…



KOH or KNO_3

$Fe(OH)_3$ or $Fe(NO_3)_3$

We know that KOH and $Fe(NO_3)_3$ don't form…that's what we started with.

What about KNO_3 and $Fe(OH)_3$?

What about KNO_3 and $Fe(OH)_3$?

They are both possible products of the reaction. Could they both form? Which one forms first? Do they form together? How would you know?

$$K_{sp}$$

Precipitation is just the reverse of dissolution.

Solubility Product:

$Ba_3(PO_4)_2$ (s)$\leftrightarrow$ 3 Ba^{2+} (aq) + 2 PO_4^{3-}(aq)

Precipitation:

3 Ba^{2+} (aq) + 2 PO_4^{3-}(aq) $\leftrightarrow$ $Ba_3(PO_4)_2$ (s)

It’s just K “upside down”

When you have 2 possible reactions…

BIGGEST K wins!

Or, in this case, SMALLEST K_{sp}

$K_{precipitation} = 1/K_{sp}$

Small K_{sp} means big $K_{precipitation.}$

Precipitation is just the reverse of dissolution.

KNO_3 (s) $\leftrightarrow$ K^+ (aq) + NO_3^-(aq)

K_{sp} (KNO_3) = HUGE (K$^+$ salts are very soluble and nitrates are very soluble)

$Fe(OH)_3$ (s) $\leftrightarrow$ Fe^{3+} (aq) + 3 OH^-(aq)

K_{sp} ($Fe(OH)_3$)=2.79×10^{-39}

So the only reaction to consider is…

$Fe(OH)_3$ (s) $\leftrightarrow$ Fe^{3+} (aq) + 3 OH^-(aq)

K_{sp} ($Fe(OH)_3$)=2.79×10^{-39}

All equilibrium problems have 3 parts…yada yada yada…

K_{sp} ($Fe(OH)_3$)=$2.79x10^{-39}$

$Fe(OH)_3$ (s) ↔ Fe^{3+} (aq) + 3 OH^{-}(aq)

I

C

E

K_{sp} = $2.79x10^{-39}$ = $[Fe^{3+}][OH^{-}]^3$

What do we know?

Don't forget the dilution

500 mL of 0.100 M $Fe(NO_3)_3$ is mixed with 250 mL of 0.100 M KOH.

So…

Dilution is the solution!

0.100 M x 0.500 L = 0.05 mol/0.750 L = 0.0667 M

0.100 M x 0.250 L = 0.025 mol/0.750 L = 0.0333 M

K_{sp} $(Fe(OH)_3)$=2.79x10^{-39}

$Fe(OH)_3$ (s) ↔ Fe^{3+} (aq) + 3 OH^-(aq)

I	-	0.067	0.033
C	-	-x	-3x
E	-	0.067-x	0.033-3x

K_{sp} = 2.79x10^{-39} = $[0.067\text{-}x][0.033\text{-}3x]^3$

This is an algebraic mess BUT…K is really small.

K is really small…

…which means… $Fe(OH)_3$ is not very soluble.

So, x is going to be huge! We can use that to our advantage.

We can mathematically precipitate out ALL of the $Fe(OH)_3$ and then redissolve it!

What is rate limiting?

$Fe(OH)_3\ (s) \leftrightarrow Fe^{3+}\ (aq) + 3\ OH^-(aq)$

I	-	0.067	0.033
C	-	-x	-3x
E	-	0.067-x	0.033-3x

0.067-x = 0

X = 0.067

0.033 – 3x = 0

X=0.011

The hydroxide runs out first!

Double your ICE, double your pleasure!

$Fe(OH)_3$ (s) ↔ Fe^{3+} (aq) + 3 OH^-(aq)

I	-	0.067	0.033
C	+0.011	-0.011	-3*(0.011)
I	0.011	0.056	0
C	-x	+x	+3x
E	0.011-x	0.056+x	3x

$K_{sp} = 2.79x10^{-39} = [0.056+x][3x]^3$

Look how much simpler that is. Even better, let's try and solve it the easy way!

Double your ICE, double your pleasure!

$K_{sp} = 2.79x10^{-39} = [0.056-x][3x]^3$

Assume x<<0.056!

$2.79x10^{-39} = [0.056][3x]^3 = 0.056*27x^3$

$1.8452x10^{-39} = x^3$

$1.23x10^{-13} = x$!

Pretty good assumption.

Double your ICE, double your pleasure!

	$Fe(OH)_3$ (s) ↔	Fe^{3+} (aq) +	3 OH^-(aq)
I	-	0.067	0.033
C	+0.011	-0.011	-3*(0.011)
I	0.011	0.056	0
C	- $1.23x10^{-13}$	+ $1.23x10^{-13}$	+3($1.23x10^{-13}$)
E	0.011	0.056	$3.68x10^{-13}$

0.011 M $Fe(OH)_3$ precipitate

0.011 M $Fe(OH)_3$*0.75 L * 106.9 g/mol = 0.9 g $Fe(OH)_3$

Neat trick, huh?

Actually, that is another little trick in your ICE arsenal…

We know what to do when x is small. Now, if we suspect x is large, we can try this little trick.

In fact, you could always forcibly do a reaction to change the initial condition. After all, in the end the equilibrium will decide where it finishes.

Clicker question

If you have 500.0 mL of a solution that is 0.022 M in Fe^{2+} and 0.014 M in Mg^{2+} and add 10.00 mL of 0.100 M K_2CO_3. What is left in solution after the precipitation?

$K_{sp}(FeCO_3) = 3.07 \times 10^{-11}$

$K_{sp}(MgCO_3) = 6.82 \times 10^{-6}$

A. 1.56×10^{-9} M CO_3^{2-}, 0.0196 M Fe^{2+}, 0.0137 M Mg^{2+}

B. 0.078 M CO_3^{2-}, 3.9×10^{-10} M Fe^{2+}, 0.014 M Mg^{2+}

C. 0.086 M CO_3^{2-}, 0.022 M Fe^{2+} , 7.9×10^{-10} M Mg^{2+}

D. 5.82×10^{-4} M CO_3^{2-}, 0.0216 M Fe^{2+}, 0.0117 M Mg^{2+}

If you have 500.0 mL of a solution that is 0.022 M in Fe^{2+} and 0.014 M in Mg^{2+} and add 10.00 mL of 0.100 M K_2CO_3. What is left in solution after the precipitation?

Dilution:

$$0.022\ M \times \frac{500\ mL}{510\ mL} = 0.0216\ M\ Fe^{2+}$$

$$0.014\ M \times \frac{500\ mL}{510\ mL} = 0.0137\ M\ Mg^{2+}$$

$$0.100\ M \times \frac{10\ mL}{510\ mL} = 0.00196\ M\ CO_3$$

$K_{sp}(FeCO_3) = 3.07\text{x}10^{-11}$

$K_{sp}(MgCO_3) = 6.82\text{x}10^{-6}$

I expect the $FeCO_3$ to precipitate first

$$FeCO_3(s) \leftrightharpoons Fe^{2+}(aq) + CO_3^{2-}(aq)$$

		0.0216		**0.00196**
		-x		-x
		0.0216-x		0.00196-x

$$3.07 \times 10^{-11} = (0.0216-x)(0.00196-x)$$

Assume x is LARGE!

$FeCO_3(s) \leftrightharpoons Fe^{2+}(aq) + CO_3^{2-}(aq)$

		0.0216		0.00196
		-0.00196		-0.00196
		0.0196		0
		+x		+x
		0.0196+x		x

$3.07x10^{-11} = (0.0196+x)(x)$

Assume x <<0.0196

$3.07x10^{-11} = 0.0196x$

$X=1.56x10^{-9}$

Check $MgCO_3$ equilibrium

After the $FeCO_3$ precipitates, the CO_3^{2-} concentration is only 1.56×10^{-9}

$K_{sp}(MgCO_3) = 6.82 \times 10^{-6}$

$Q_{sp} = (1.56 \times 10^{-9})(0.0137\ M) = 2.137 \times 10^{-11}$

Q << K, so no $MgCO_3$ precipitates!

CB: For the equilibrium

$Ca(OH)_2(s) \leftrightarrow Ca^{2+}(aq) + 2OH^{-}(aq)$

decreasing the pH would:

A Decrease solubility

B Increase solubility

C Have no effect on solubility

CB: For the equilibrium

$PbBr_2(s) \leftrightarrow Pb^{2+}(aq) + 2Br^-(aq)$

Adding NaBr(s) would:

A Decrease solubility

B Increase solubility

C Have no effect on solubility